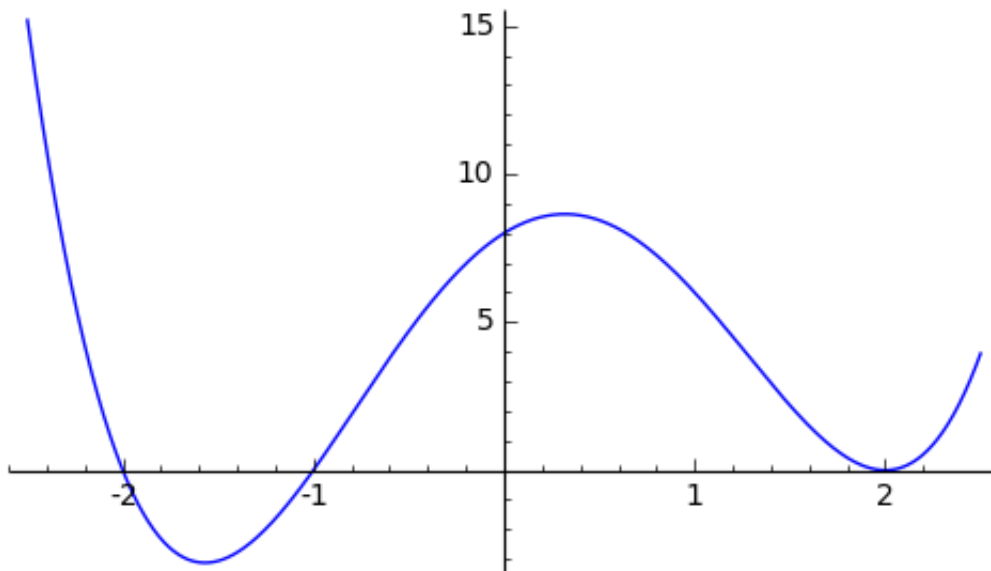


Advanced ODE Tutorial

Plot a function of one variable

```
var("x")
f(x) = (x+2)*(x+1)*(x-2)^2
plotf = plot(f(x), (x,-2.5,2.5), color='blue', linestyle='-',
thickness=1)
show(plotf, figsize=[5,3])
```



Taylor polynomials

```
var("a,t")
f(t) = tanh(a*t)/(1+t^2)
p30(t) = taylor(f(t),t,0,3); show(p30(t))
```

$$-\frac{1}{3} (a^3 + 3a)t^3 + at$$

Eigenvalues/vectors

```
A = matrix([[0,1],[-4,-5]])
show(A.eigenvalues())
```

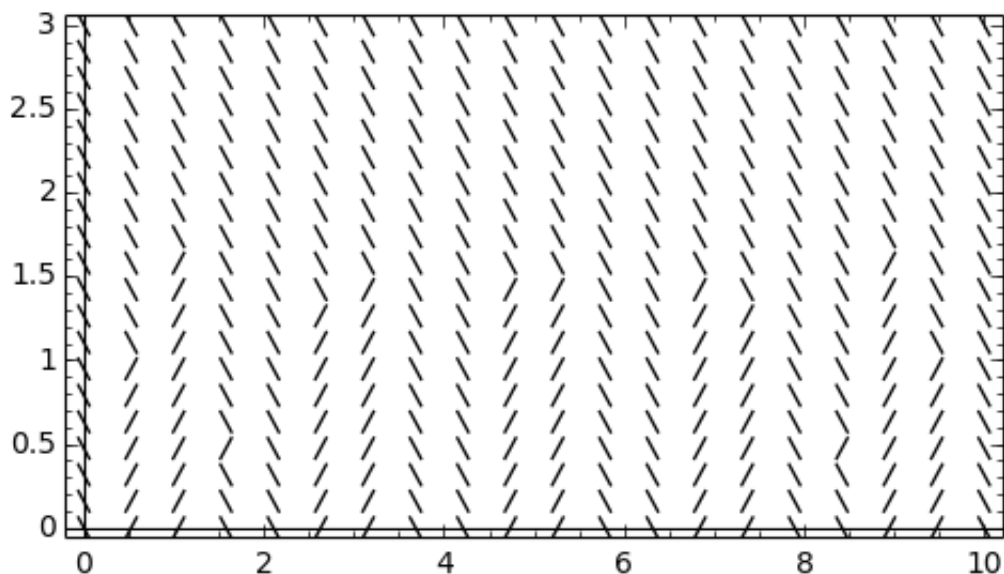
$$[-1, -4]$$

```
A = matrix([[0,1],[-4,-5]])
show(A.eigenvectors_right())
```

$$[(-1, [(1, -1)], 1), (-4, [(1, -4)], 1)]$$

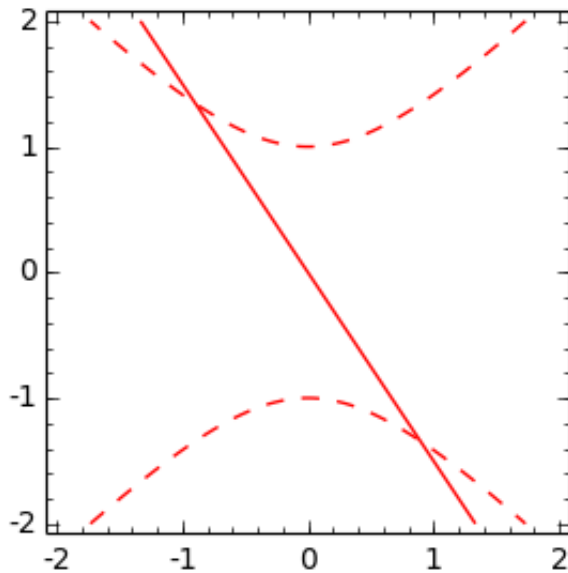
Plot slope field

```
var("t,x")
f(x,t) = x*(1-x) - cos(pi*t)
n(x,t) = sqrt( f(x,t)^2 )
plotsf = plot_slope_field(f(x,t)/n(x,t),(t,0,10), (x,0,3))
show(plotsf, figsize=[5,3])
```



Plot nullclines

```
var("x,y")
f(x,y) = x^2 - y^2 + 1
g(x,y) = 3*x + 2*y
plotf = contour_plot(f(x,y), (x,-2,2), (y,-2,2), contours=[0],
linestyles='--', linewidths=1, fill=False, cmap='hsv')
plotg = contour_plot(g(x,y), (x,-2,2), (y,-2,2), contours=[0],
linestyles='-', linewidths=1, fill=False, cmap='hsv')
plotall = plotf + plotg
show(plotall, figsize=[5,3])
```

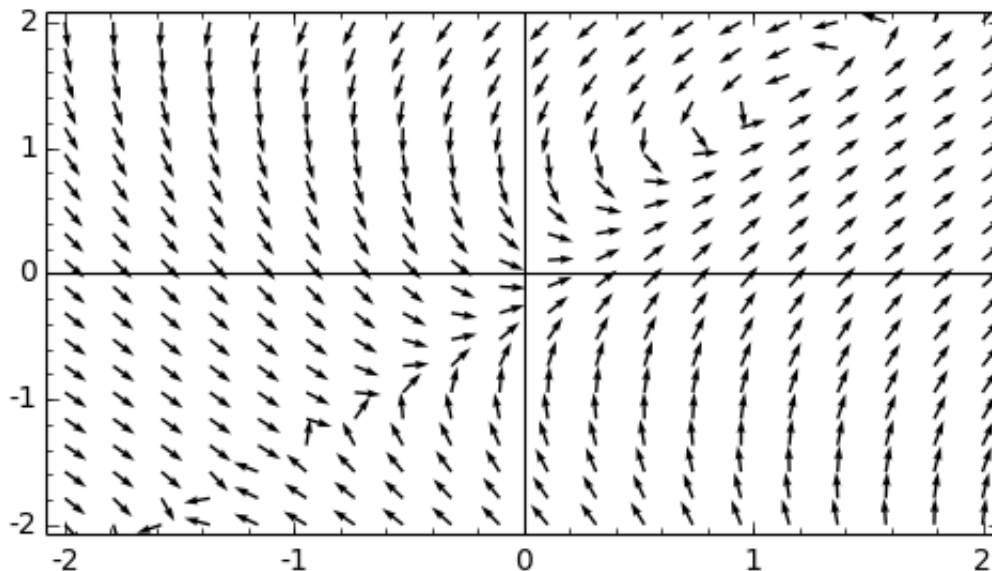


Plot vector field

```

var("x,y")
f(x,y) = x^2 - y^2 + 1
g(x,y) = 4*x - 3*y
n(x,y) = sqrt( f(x,y)^2 + g(x,y)^2 )
plotvf = plot_vector_field((f(x,y)/n(x,y),g(x,y)/n(x,y)),(x,-2,2),
(y,-2,2))
show(plotvf, figsize=[5,3])

```



Plot nullclines and vector field

```

var("x,y")
f(x,y) = x^2 - y^2 + 1
g(x,y) = 4*x - 3*y
n(x,y) = sqrt( f(x,y)^2 + g(x,y)^2 )

```

```
plotf = contour_plot(f(x,y), (x,-2,2), (y,-2,2), contours=[0],  
linestyles='--', linewidths=1, fill=False, cmap='hsv')  
plotg = contour_plot(g(x,y), (x,-2,2), (y,-2,2), contours=[0],  
linestyles='-', linewidths=1, fill=False, cmap='hsv')  
plotvf = plot_vector_field((f(x,y)/n(x,y),g(x,y)/n(x,y)),(x,-2,2),  
(y,-2,2))  
plotall = plotf + plotg + plotvf  
show(plotall, figsize=[5,3])
```

